Bankers and regulatory agencies use limits on the size of individual credits as a device to control the risk of the loan portfolio. A natural extension of this approach is to set limits on the overall concentration of the loan portfolio. The concentration ratio presented here provides a measure of portfolio concentration and can serve as a useful tool for managing the risk of the loan portfolio.

Degree of diversification is one of the important elements that determine the risk of a commercial loan portfolio. The loan portfolio of a New England bank is presented in Figure 1. How diversified is the portfolio? On the one hand, it seems very well diversified with $1.14 billion in loans spread over 4,460 firms. On the other hand, most of the total value of the portfolio is concentrated in just a few loans. For instance, 61% of the value of the portfolio is in the 4.3% of loans greater than $5 million. These figures suggest that any evaluation of portfolio diversification must consider not only the number but also the size of the loans. Bank officers can use this concentration ratio to determine the effect of a new loan on portfolio diversification, to calculate the ideal loan size for increasing diversification, and to estimate the largest loan that will not increase concentration.

Portfolio Weights

The return of a portfolio is simply a weighted average of the returns of the individual loans. For instance, assume a $10,000 portfolio has an $800 loan paying 7%, a $2,700 loan earning 8%, and a $6,500 loan yielding 9%. As shown below, the total interest income of the portfolio is $857.

\[
\text{Portfolio Interest} = (7\%)(800) + (8\%)(2,700) + (9\%)(6,500) \\
= 56 + 216 + 585 \\
= 857
\]

The overall earnings rate of the portfolio is ($857/$10,000), or 8.57%. This same result follows from weighting the return of each individual loan by its importance in the portfolio. The weight for each loan is its value divided by the $10,000 total value of the portfolio. The appropriate weights in this example are 8%, 27%, and 65%.

\[
\text{Portfolio Return} = (8\%)(7\%) + (27\%)(8\%) + (65\%)(9\%) = 8.57\%
\]

This measure of the portfolio return means that the average dollar in the portfolio earns at a rate of 8.57%.

The Concentration Ratio

The weights derived above show the concentration of portfolio value in each of the loans. For example, 65% of the total value of the portfolio is concentrated in the largest loan of $6,500. A measure of the overall concentration of the portfolio (C) is a weighted average of the concentrations of the individual loans.

© 1998 by RMA. Ford is Nicolas Salgo Professor of Business Administration, University of Maine, Orono, Maine.
The preceding equation shows that the concentration ratio is the sum of the squares of the concentrations of the individual loans. This concentration ratio means that the average dollar in the portfolio is in a loan that represents 50% of the total value of the portfolio.

The possible values of the concentration ratio vary from zero for a well diversified portfolio to 100% for a portfolio of one loan. The lower the ratio, the less concentrated and the more diversified the portfolio. A portfolio with a concentration ratio of 5% has twice the diversification of a portfolio with a ratio of 10%.

The concentration ratio allows loan officers to estimate the diversification effects of both the number and the size of loans. If the example $10,000 portfolio had three loans of $3,333 each, then the concentration ratio would be 33%.

The effect of the large loan is to increase the portfolio concentration from 33% to 50%. The magnitude of the increase is always (CN)—the concentration ratio times the number of loans. In this case the result is (50%)(3), or 1.5. The effect of having large loans is to increase the concentration ratio by a factor of 1.5 from 33% to 50%.

The Diversification Quotient

If the portfolio consists of (N) loans of equal size, then the concentration ratio is (1/N). With a concentration ratio of 50%, the example portfolio has

the same diversification as a portfolio of (1/50%) or two loans of $5,000 each.

This analysis suggests another approach to interpreting the meaning of the concentration ratio. The reciprocal of the concentration ratio indicates that the portfolio has the same degree of diversification as a portfolio of (1/C) loans of equal size. A logical name for this reciprocal is the diversification quotient: Concentration and diversification are reciprocal expressions of the same concept.

The diversification quotient makes more sense intuitively than the concentration ratio. It is quite natural to think of diversification in terms of the number of independent loans in the portfolio. Reporting a concentration ratio of 0.83% carries less meaning than reporting a diversification quotient of (1/0.83%) or 120. Both figures convey the same information, but it is easier to understand that the portfolio has the same diversification as a portfolio of 120 loans of equal size.

Portfolio Management

One of the important conclusions of modern financial theory is that investors should reduce risk by holding a diversified portfolio of assets. An implication of this conclusion is that investors should evaluate a potential new investment not in isolation but in terms of its impact on their portfolios. The concentration ratio allows bankers to gauge the effect of a new credit on the diversification of the loan portfolio.

Assume a new loan of $5,000 is added to the existing $10,000 portfolio of three loans. The total portfolio value is now $15,000 and the weight of the new loan is ($5,000/$15,000) or 33%. The weights of the other loans are now 5%, 18%, and 43%. The calculation below shows that the new concentration ratio of the portfolio is 33%.

$$C = (5\%)(5\%) + (18\%)(18\%) + (33\%)(33\%) = 33\%$$
The effect of adding the new loan is to reduce the concentration ratio from 50% to 33%. The change in diversification is apparent in the increase of the diversification quotient from two to three. The portfolio now has the same diversification as a portfolio of three loans of equal size.

For this small portfolio, it is easy to calculate the diversification effects of various loan sizes using a spreadsheet. Figure 2 shows that adding a $5,000 loan to the existing $10,000 portfolio produces the greatest reduction in concentration. The general result is that adding a loan equal to the concentration ratio times the value of the portfolio produces the greatest reduction in concentration. In this case, multiplying the current concentration ratio of 50% and the current portfolio value of $10,000 indicates that the most efficient loan size is $5,000. Another general result is that adding a loan of the efficient size always increases the diversification quotient by one. In this case, adding a $5,000 loan increases the diversification quotient from two to three.

For loan sizes above $5,000, the concentration effect of a large dollar value begins to outweigh the diversification effect of adding a new loan. Figure 2 shows that $20,000 is the largest loan size that will not increase the concentration of the portfolio. This maximum loan size is always the portfolio value (V) times twice the concentration ratio (C) divided by one minus the concentration ratio.

\[ V \left[ \frac{2C}{1-C} \right] = 10,000 \left[ \frac{2(50\%)}{1-50\%} \right] = 20,000 \]

For larger portfolios, it is easier to use a formula to calculate the change in the concentration ratio that results from a new loan. Adding a $5,000 credit to the $10,000 portfolio produces a new portfolio with two parts: the new $5,000 with a 100% concentration in the new loan and the old $10,000 with a 50% concentration. Combining these two parts dilutes their individual concentrations. For example, the old $10,000 portfolio is now only 67% of the new $15,000 portfolio. Its concentration is diluted to 67% of 50% or only 33% in the new portfolio.

The mixing of liquids provides a familiar example of dilution. Combining five gallons of water and five gallons of orange juice produces a 10-gallon mixture with a 50% concentration of orange juice. Adding an additional 5 gallons of water dilutes the concentration of orange juice to (5/15), or 33%. As shown below, the concentration ratio of the new portfolio is the weighted average of the concentration ratios of the components.

\[ C = \frac{(33\%) \cdot [(33\%)(100\%)] + (67\%) \cdot [(67\%)(50\%)]}{(33\%)^2 (100\%) + (67\%)^2 (50\%) = 33\%} \]

This formula suggests managing the overall diversification of the commercial loan portfolio of a large institution by assigning concentration objectives to each lending unit. For example, suppose that a bank has three regional units with total loans of $80 million, $140 million, and $180 million and concentration ratios of 5%, 10%, and 20%, respectively. As shown below, the concentration ratio of the overall portfolio is 5.5%, the weighted average of the concentration ratios of the three units. In each case, the appropriate weight is the value of loans in the region divided by the $400 million value of the entire portfolio.

\[ C = (20\%)^2 (5\%) + (35\%)^2 (10\%) + (45\%)^2 (20\%) = 5.5\% \]

The bank can control its diversification by changing the concentration objectives or the amount of investment in each of the regional subportfolios.

**Conclusion**

Bankers and regulatory agencies use limits on the size of individual credits as a device to control the risk of the loan portfolio. A natural extension of this approach is to set limits on the overall concentration of the loan portfolio. The concentration ratio provides a measure of portfolio concentration and serves as a useful tool for managing the risk of the loan portfolio. The
concentration ratio is very easy to calculate with a spreadsheet as simply the sum of the squared weights of each loan in the portfolio. It can also be used to calculate the concentration of the portfolio by industry or geographic region. For example, the industry concentration of the portfolio can be calculated using the total value of loans in each industry as the appropriate weights. Obviously the concentration ratio does not deal with all facets of diversification. The ratio does, however, convey valuable information about an important portfolio characteristic at virtually no cost.