How to Choose Interest Rate Models in the QRM System

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Quantitative risk management is widely used in large banks for asset/liability management purposes. Version 8.14 of the QRM system offers three models to use in measuring interest rate movements: Black-Derman-Toy (BDT), Black Karasinski (BK), and Hull-White (HW). The OCC and the Fed require banks to document their model testing to demonstrate prudence in model selection. This article shows how to choose among different interest rate models and provides testing results.

Effective use of a quantitative risk management comes with a better understanding of the three models currently available to measure interest rate risk. Each has its plusses. After an analysis performed at KeyCorp, one emerged as the winner for this institution.

Black Karasinski. The BK model assumes that the short-term interest rate $r$ follows the lognormal distribution. The process can be described as follows:

$$d(\log r) = f(t)(\log m(t) - \log r)dt + s(t)dz$$

where $m(t)$ is the target rate, $f(t)$ is mean reversion factor, and $s(t)$ is the local interest rate volatility. The BK model uses $m(t)$, $f(t)$, and $s(t)$ as the three inputs to match the three features of the term structure of the real-world yield curve, volatility curve, and cap curve.

The advantage of the BK model is that each of these features is observable, while some features may not be observable in, says, the HW model.

Hull-White. The HW model is the extended Vasicek model, for which short-term interest rates follow a normal distribution. The process can be described as follows:

$$dr = f(t)(m(t) - r)dt + s(t)dz$$

where $m(t)$ is the target rate, $f(t)$ is mean reversion factor, and $s(t)$ is the local interest rate volatility. The HW model uses $m(t)$, $f(t)$, and $s(t)$ as the three inputs to match the three features of the term structure of the real world, in this case, yield curve, volatility curve, and future local volatility.

Black-Derman-Toy. The BDT model develops a binomial model that uses traded prices as given and calculates a lognormal short-term interest rate to fit the yield curve and volatility curve.

The three models all have sound theoretical foundations. However, each model has different
sensitivity to changes in the yield volatility structure and yield curve term structure. For the asset and liability management purpose, it’s important to keep the value of equity relatively stable over time. Therefore, the model selected should be the least sensitive to changes in the yield volatility structure and yield curve term structure.

We chose to test the products within which the cash flows are rate-path dependent but are without prepayment options. This made it possible to avoid the complication of separating the effects of model sensitivity and prepayment. The products that fit those criteria are:

- LIBOR Caps
- Callable Fixed-Rate Debt
- Indexed Amortizing Swaps

Yield volatility structure scenarios. We performed tests on five different scenarios of yield volatility structure:

1. Base case (current volatility structure from QRM).
2. The volatility is reduced by half for all terms.
3. The volatility is doubled for all terms.
4. The volatility is flat for all terms (the middle point of the base case is chosen in this case).
5. The slope of the volatility curve increases from the base case.

Yield curve term structure scenarios. We chose to have three different scenarios on yield curve term structure:

1. Base case (current yield curve from QRM).
2. A flat yield curve regardless of term (the middle point of the base case is chosen in this case).
3. A downward sloping yield curve for an extreme yield curve case.

We believe that the above scenarios cover most of the dramatic changes in the yield volatility structure and the yield curve term structure. If an interest rate model shows stability for all the scenarios, the model results should be stable most of the time and should be chosen in the production process.

We also tested the three different interest rate models by product type for all of the scenarios. It is found that the BDT model is highly sensitive to yield volatility structure changes regardless of product type (Please see Figure 1 and Figure 2 for the test results on LIBOR Caps).

Both the BK and the HW have low sensitivities to yield volatility and yield curve term structure changes. As a matter of fact, the results of the BK model are marginally less sensitive than those of the HW models, although both models show high stability for different products and different scenarios.

Note that the lower the sensitivity-to-yield structure and yield-volatility structure changes, the better it is for the QRM production process. Therefore, we recommend the BK (or HW) model over the BDT model. 

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