Market risk-potential loss in market value of a trading position—is determined by two factors: market risk exposure and volatility of the markets.

- The exposure ("Greeks") demonstrates a vulnerability of trading positions to the market changes. It is expressed as the market value change for a small change in relevant market factors. There exist a large number of these measures that describe the market value sensitivity to various market factors, such as interest rates, foreign exchange rates, volatilities, and so on, and their combinations. Most of the tools in market risk management are exposure-based.

- With all their usefulness for hedging and scenario-based risk analysis, these measures describe only trading positions and do not reflect how risk of the same position changes with changes in the markets volatility. VAR, on the other hand, is the only risk tool that combines exposure of trading positions with measurement of volatility of current market conditions into a single risk measure-potential loss. The significance of VAR as a market risk tool makes accuracy essential. If we do not know whether our risk measure parallels reality, none of the other issues, including VAR implementation challenges or its computational efficiency, are of any importance. The only valid criterion of accuracy is that VAR realistically measures what it is supposed to measure-maximum market loss for a given holding period and confidence level. A backtesting procedure performs this analysis by comparing each day’s VAR to actual losses. From a statistical point of view, the backtesting procedure evaluates accuracy of our P&L distribution forecast (at the selected point corresponding to the required confidence level) by comparing it with a “real” distribution of profits and losses.

The actual task of statistical measurement of market risk is broader than calculating VAR for individual positions and their portfolios; it involves forecasting corresponding P&L distributions. The better we can forecast the future P&L distributions, the more accurate the VAR! From that perspective, the backtesting procedure is also becoming a tool for selection of the best VAR methodology as well as for optimization of its parameters.

Producing P&L distribution begins with the distributions of market factors relevant for trading positions. This problem is the most important theoretical step of VAR calculations and it deserves a separate discussion.

Forecasting Distribution of Market Factors

There presently is no general theory to help in solving the problem of forecasting except empirical analysis, which does uncover certain helpful facts about the nature of the distributions. One of the most important facts is that market volatility usually is relatively stable within a short time horizon and changes over the long term. Called “quasi-stability of the distributions,” this forms the basis for a statistical approach to market risk measurement. In terms of risk measurement, quasi-stability means that:

- Market volatility in the immediate future will be essentially the same as in the recent past.
- Past distributions of changes in market factors will most likely maintain their shape in the near future.
- Their historical distributions can be used as a predictor of the probability distribution in the future.
All VAR calculations in all VAR models are based on this fact. The most significant differences between the models relate to specific techniques used to analyze historical behavior of market factors and specific assumptions about shapes of the distributions.

Optimal Look-Back Period

Commonality of the methodology leads to commonality of issues among all VAR models. The most significant issue is how much historical data (look-back period) to use in a VAR calculation to produce the most accurate risk assessment. Accuracy declines if we use too many or too few data. It might sound strange for statistical calculations, but, in fact, it is possible to have too many historical data if the process to be modeled (markets volatility) varies significantly with time. In this instance, data from the distant past might have no relevance to current market conditions, thus reducing accuracy of the calculations. On the other hand, too few data also will produce statistically unreliable results!

Backtesting is used to identify the appropriate length of time. We can backtest several models with different look-back intervals. The model that produces the most accurate VAR will be the one using the optimal look-back period.

Popular Models for Distributions Forecasts

The last issue to clarify is the shape of the distributions. Parametric methods (the variance-covariance and Monte Carlo) assume that the distributions have a specific functional form (most often the normal distribution is used) and use historical data just to determine parameters of the distributions. The major advantage of this approach is that it significantly simplifies the calculations while producing a seemingly good fit to the distributions of past data.

Figure 1 illustrates the discussion. It presents a typical example of the frequency distribution for changes in market values. In its middle section, the distribution has a characteristic bell-shaped profile that can be easily approximated by the normal distribution. However, comparison of the two graphs also reveals that the normal distribution is not a good approximation to the actual distribution for large price changes; the large price movements were observed much more frequently than is suggested by the normal distribution model (fat tails). Since risk management is mostly concerned with measurement of large losses related to high confidence levels, modeling these events is of primary interest.

Historical simulation methodology is based on the assumption that future distributions have exactly the same shape as past distributions. The assumption is reasonable. If we know only that future distribution should be the same as in the past, then past distribution itself is the best model for future distribution. From this point of view, the historical simulation should be inherently the most accurate method among other VAR approaches.

Example of VAR Calculation in Historical Simulation

Let’s calculate a one-day VAR for a hypothetical bond to present the methodology. Since the bond price depends on its yield, the first task is to forecast the distribution of the yield’s future changes. Figure 2 provides an example of these calculations.

Column 1 assumes a 101-day look-back period. The second column presents 100 daily changes of the yield. The numbers describe past volatility of the factor. Historical simulation models the future probability distribution of the yield changes simply by assuming that it is the same as the past distribution. In other words, we assume that each of the yield changes from the second column can occur in the future. An advantage of this method of modeling the future
distributions is that it automatically includes all the features of actually observed distributions, such as bell-shaped distribution, typical for small and medium-sized changes, and fat-tails distribution, for largest changes in market factors.

**Forecasting P&L Distribution**

The next step is to produce a distribution of bond market value changes from the yield changes distribution. To simplify calculations, the example assumes that the bond market value changes proportionally to yield change and its value declines by $1 for each basis point of yield increase. The distribution of the bond value changes now can be produced by multiplying each yield change from the derived yield distribution by a factor of -$1/bp (Figure 2, column 3).

For linear instruments, distributions of changes in market values are just scaled-up or scaled-down distributions of the underlying market factors. There are many examples of this relationship, including noncontingent financial instruments, such as equity, FX spot, commodity, and low-convexity interest rates instruments. In the more general case of value-market factor, the relationship is complicated by nonlinearity. The nonlinear component of price-market factor relationship is typical for option-related or convex instruments.

The sensitivities-based method described above can be used to generate distributions for both types of products. Its major advantage is that it is fast. Limits of applicability of the method are determined by accuracy of the linear approximation.

For instances in which the sensitivities-based approximation is not applicable, historical simulation uses a so-called full valuation method—another very popular method by which to derive P&L distributions. Full valuation method produces the distributions of changes in market values using pricing algorithms of corresponding instruments. This method produces the most accurate assessment of the distributions but with the cost of relatively longer processing time.

**Portfolio Risk**

To determine the portfolio risk of instruments, it’s necessary to determine the distribution of changes in their market value. In the historical simulation approach, this distribution is determined by aggregating the distributions of changes in market values of components of the portfolio. The distributions aggregation procedure is based on the fact that a change in portfolio market value for each day of the selected look-back period is given by the sum of market values changes for all its instruments and positions for the same day.

This aggregation procedure is applicable to a portfolio with any number of instruments, a very attractive feature of the methodology. This is because alternative VAR methodologies that are based on a variance-covariance matrix are much more restrictive concerning the number of market factors used. Practically speaking, the limit of other methods is about 700-800 various factors. The historical simulation, however, can calculate risk for portfolios of 3,000 to 5,000 factors within a few seconds on a regular personal computer. Some VAR applications use up to 24,000 various market factors!

Another attractive feature of historical simulation is that the approach produces the P&L portfolio distribution, which explicitly incorporates all the statistical properties of its components, including fat tails and correlations. As a result, historical simulation performs a very accurate evaluation of all market value distributions on all levels of aggregation from individual instruments to total portfolio.

Finally, this aggregation procedure has a very clear interpretation. The derived portfolio distribution describes actual daily changes in market value of the portfolio that the portfolio manager would have observed if the positions
were established 101 days ago. This interpretation makes the historical simulation a very transparent risk measurement methodology that allows users not only to quantify market risk but also to understand the results of the risk measurement process.

**From P&L Distribution to VAR**

The last step in the VAR process is to measure VAR from the derived distributions of changes in market values of individual instruments and their portfolios. According to the definition of VAR, measuring VAR requires determining loss level for a given level of confidence. Given the derived P&L distribution, historical simulation calculates VAR by numerically searching for the point on the distribution that corresponds to a required confidence level.

Calculations are very intuitive and simple. For example, calculating VAR for a confidence level of 95% requires finding an event with a loss of such a magnitude that the number of all the losses exceeding this one will represent no more than 5% of all the events. VAR would be measured by this event's loss.

This method is universal since it can be used to measure risk for P&L distribution of any shape and form. That feature is especially important if we wish to include both “fat tails” and options with asymmetrical payoffs in the risk measurement process. Furthermore, relating VAR to the loss of a specific point of the P&L distribution historical simulation also facilitates a greater transparency of risk management process, because it enhances understanding of the current risk level as related to specific historical market moves.